[4] P. Rizzi, *Microwave Engineering Passive Circuits*. Englewood Cliffs, NJ: Prentice-Hall, 1988, ch. 4.
[5] R. E. Collin, *Foundations for Microwave Engineering*, 2nd ed. New York: McGraw-Hill, 1992, ch 5.
[6] W. T. Joines and J. R. Griffin, "On using the Q of transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 258–260, Apr. 1968.
[7] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks, and Coupling Structures*. New York: McGraw-Hill, 1964.

Circuit Theory for Spatially Distributed Microwave Circuits

Ahmed I. Khalil and Michael B. Steer

***Abstract*— A spatially distributed radio-frequency (RF) circuit, microwave, or millimeter-wave circuit does not have a global reference node as required in conventional nodal analysis. Instead, local reference nodes associated with ports are required. This paper adapts modified nodal analysis to accommodate spatially distributed circuits, allowing conventional harmonic balance and transient simulators to be used.**

***Index Terms*— Circuit simulation, computer-aided design, microwave circuits.**

I. Introduction

Nodal analysis is the mainstay of circuit simulation. The basis of the technique is relating nodal voltages (voltages at nodes referenced to a single common reference node) to the currents entering the nodes of a circuit. Generally, the art of modeling is then to develop a current/nodal-voltage approximation of the physical characteristics of a device or structure. With spatially distributed structures, a reasonable approximation can sometimes be difficult to achieve. The essence of the problem is that a global reference node cannot reasonably be defined for two spatially separated nodes when the electromagnetic field is transient or alternating. In this situation, the electric field is nonconservative and the voltage between any two points is dependent on the path of integration and, hence, voltage is undefined. This includes the situation of two separated points on an ideal conductor. In a time-domain context, it takes a finite time for the state at one of the points on the ideal conductor to affect the state at the other point. In the case of waveforms on digital interconnects, this phenomenon has become known as retardation [1]. With high-speed digital circuits, it is common to model ground planes by inductor networks so that interconnects are modeled by extensive meshes of resistors, inductors, and capacitors. Consequently, no two separated points are instantaneously coupled. In transient analysis of distributed microwave structures, lumped-circuit elements can be embedded in the mesh of a time discretized electromagnetic-field solver such as a finite-difference time-domain (FDTD) field modeler [2], [3]. The temporal separation of spatially distributed points is then inherent to the discretization of the mesh.

Manuscript received December 2, 1997. This work was supported by the Defense Advanced Research Projects Agency (DARPA) under the MAFET Thrust III Program as DARPA Agreement DAAL01-96-K-3619.

The authors are with the Electronics Research Laboratory, Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27695-7914 USA.

Publisher Item Identifier S 0018-9480(98)07235-4.

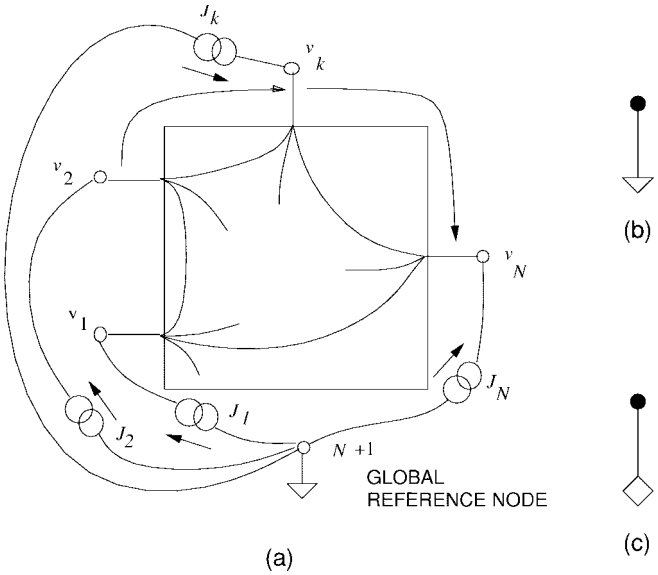

Fig. 1. Nodal circuits. (a) General nodal circuit definition. (b) Conventional global reference node. (c) Local reference node proposed here.

With a frequency-domain electromagnetic-field simulator, ports are defined and, thus, a port-based representation of the linear distributed circuit is produced. With ports, a global reference node is not required. Instead, a local reference node (one of the terminals of the two-terminal port) is implied. The beginnings of a circuit theory incorporating ports in circuit simulation has been described and termed the compression matrix approach [4], [5]. This milestone work presented a technology for integrating port-based electromagnetic-field models with nonlinear devices. Circuit simulation using port representation has been reported in [6]. This requires the representation of nodally defined circuits in its port equivalent by a general-purpose linear multiport routine. Hence, the advantage of accessing information at all nodes, as in nodal analysis, is lost.

The purpose of this paper is to extend the circuit theory behind the compression matrix approach to general-purpose circuit simulators based on nodal analysis. In particular, we present the concept of local reference nodes that enables port-based network characterization to be used with nodally defined circuits in the development, by inspection (the preferred approach), of what is termed a locally referenced nodal admittance matrix. A procedure for handling and moving the local reference nodes is described, along with circuit-reduction techniques that facilitate efficient simulation of nonlinear microwave circuits.

II. Nodal-Based Circuit Simulation

The most popular method for circuit analysis in the frequency domain is the nodal admittance matrix method. In the nodal formulation of the network equations, a matrix equation is developed that relates the unknown node voltages to the external currents using the model shown in Fig. 1. All node voltages are then defined with respect to an arbitrarily chosen node called the global reference node. Eliminating the row and column associated with the global reference node leads to a definite admittance matrix, and then the solution for the node voltages is straightforward. In this type of analysis, only one reference node can exist.

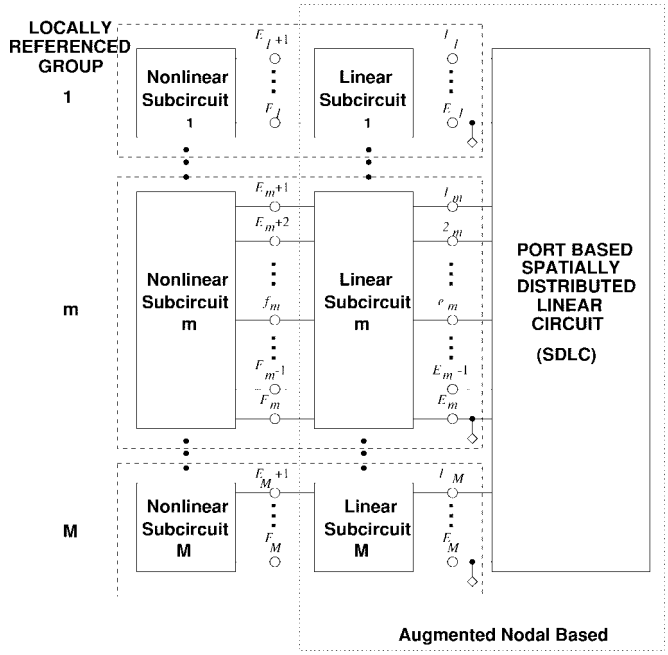

Fig. 2. Port defined system connected to nodal defined circuit.

III. Spatially Distributed Circuits

A. Port Representation

Electromagnetic structures can only be strictly analyzed using port excitations. The spatially distributed linear circuit (SDLC) consists of groups with each group having a local reference node. The scattering parameters are the most natural parameters to use with ports and their local reference nodes. They can be converted to port admittance matrix using

$$\boldsymbol{Y} = \boldsymbol{Y}_0(\mathbf{1} - \boldsymbol{Y}_0^{-1/2}\boldsymbol{S}\boldsymbol{Y}_0^{1/2})(\mathbf{1} + \boldsymbol{Y}_0^{-1/2}\boldsymbol{S}\boldsymbol{Y}_0^{1/2})^{-1}. \tag{1}$$

This is the most convenient form to use in circuit simulators. Before continuing, a distinction is required between the global reference node and the local reference nodes, where the symbols shown in Fig. 1 are adopted here. A general circuit with local reference nodes required with an SDLC and nonlinearities is shown in Fig. 2. This figure depicts the essential circuit analysis issue: integrating the representation of an SDLC with a circuit defined in a conventional nodal manner to obtain an augmented nodal-based description. The problem is how to handle the additional redundancy introduced by the local reference nodes. For locally referenced group number m, there are E_m terminals, $E_m - 1$ locally referenced ports, and one local reference node, which is designated E_m. The port-based system may be expressed as

$$[{}_p\boldsymbol{Y}][{}_p\boldsymbol{V}] = [{}_p\boldsymbol{I}]. \tag{2}$$

Where the port-based admittance matrix

$${}_p\boldsymbol{Y} = \begin{bmatrix} {}_p\boldsymbol{Y}_{\mathbf{1},\mathbf{1}} & \cdots & {}_p\boldsymbol{Y}_{\mathbf{1},\boldsymbol{m}} & \cdots & {}_p\boldsymbol{Y}_{\mathbf{1},\boldsymbol{M}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ {}_p\boldsymbol{Y}_{\boldsymbol{m},\mathbf{1}} & \cdots & {}_p\boldsymbol{Y}_{\boldsymbol{m},\boldsymbol{m}} & \cdots & {}_p\boldsymbol{Y}_{\boldsymbol{m},\boldsymbol{M}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ {}_p\boldsymbol{Y}_{\boldsymbol{M},\mathbf{1}} & \cdots & {}_p\boldsymbol{Y}_{\boldsymbol{M},\boldsymbol{m}} & \cdots & {}_p\boldsymbol{Y}_{\boldsymbol{M},\boldsymbol{M}} \end{bmatrix}$$

the port-based voltage vector

$${}_p\boldsymbol{V} = [{}_p\boldsymbol{V}_1 \quad \cdots \quad {}_p\boldsymbol{V}_m \quad \cdots \quad {}_p\boldsymbol{V}_M]^T$$

and the port-based current vector

$${}_p\boldsymbol{I} = [{}_p\boldsymbol{I}_1 \quad \cdots \quad {}_p\boldsymbol{I}_m \quad \cdots \quad {}_p\boldsymbol{I}_M]^T.$$

The submatrix ${}_p\boldsymbol{Y}_{\boldsymbol{i},\boldsymbol{j}}$ is the mutual port admittance matrix (of dimension $E_i - 1 \times E_j - 1$) between groups i and j of the SDLC, $I_i = [I_{1_i}\ I_{2_i}\ \cdots I_{E_i-1}]$ is the current vector of group i of the SDLC, and ${}_pV_j = [(V_{1_j} - V_{E_j})\ (V_{2_j} - V_{E_j}) \cdots (V_{E_j-1} - V_{E_j})]$ is the port voltage vector of group j of the SDLC. Defining the total number of ports for groups i to j of the SDLC as

$$n_j = \sum_{i=1}^{j} (E_i - 1). \tag{3}$$

${}_p\boldsymbol{Y}$ is then square of dimension $n_M \times n_M$.

B. Port to Local Node Representation

In order to use nodal analysis, the port-based system must be formulated in a nodal admittance form. Since there are M localized reference nodes, another redundant M rows and M columns can be added to the port admittance matrix such that

$$[{}_n\boldsymbol{Y}][{}_n\boldsymbol{V}] = [{}_n\boldsymbol{I}] \tag{4}$$

where the nodal admittance matrix

$${}_n\boldsymbol{Y} = \begin{bmatrix} {}_p\boldsymbol{Y} & \boldsymbol{Y}_1 \\ \boldsymbol{Y}_2 & \boldsymbol{Y}_3 \end{bmatrix}_{(n_M+M)\times(n_M+M)}. \tag{5}$$

The elements of each submatrix are given by

$$\boldsymbol{Y}_{\mathbf{1}}(\boldsymbol{r},\boldsymbol{c}) = -\sum_{j=n_{(c-1)}+1}^{n_c} {}_p\boldsymbol{Y}(\boldsymbol{r},\boldsymbol{j}), \qquad r = 1,\cdots,n_M \quad c = 1,\cdots,M$$

$$\boldsymbol{Y}_2(\boldsymbol{r},\boldsymbol{c}) = -\sum_{i=n_{(r-1)}+1}^{n_r} {}_p\boldsymbol{Y}(\boldsymbol{i},\boldsymbol{c}), \qquad r = 1,\cdots,M \quad c = 1,\cdots,n_M$$

$$\boldsymbol{Y}_3(\boldsymbol{r},\boldsymbol{c}) = -\sum_{i=n_{(r-1)}+1}^{n_r} \boldsymbol{Y}_1(\boldsymbol{i},\boldsymbol{c}), \qquad r = 1,\cdots,M \quad c = 1,\cdots,M$$

with $n_0 = 0$, n_r, n_c, $n_{(c-1)}$, and $n_{(r-1)}$ given by (3). The nodal voltage vector

$${}_n\boldsymbol{V} = [V_{1_1}\cdots V_{E_1-1}V_{1_2}\cdots V_{E_2-1}\cdots V_{1_M} \cdots V_{E_M-1}V_{E_1}V_{E_2}\cdots V_{E_M}]^T$$

and the branch current vector

$${}_n\boldsymbol{I} = [I_{1_1}\cdots I_{E_1-1}I_{1_2}\cdots I_{E_2-1}\cdots I_{1_M} \cdots I_{E_M-1}I_{E_1}I_{E_2}\cdots I_{E_M}]^T.$$

The admittance matrix $[{}_n\boldsymbol{Y}]$ is a nodal matrix and has M-dependent rows and M-dependent columns. Hence, it is an M-fold indefinite nodal admittance matrix corresponding to the M local reference nodes.

IV. Representation of Nodally Defined Circuits

Since there are no connections between the linear circuits at each group, the linear circuit at group i will have no mutual coupling with the linear circuit at group j $(i \neq j)$. The only coupling that can exist between different locally referenced groups is accounted for in the description of the SDLC. Hence, for the linear subcircuits (as in Fig. 2), all the entries in the admittance matrix are zero, except those relating the node parameters at the same group $(i = j)$. Defining

interfacing nodes as the nodes between lumped linear circuits and nonlinear circuits, a lumped linear circuit embedded at group m can be represented as

$$[\boldsymbol{Y}][\boldsymbol{V}] = [\boldsymbol{I}] \tag{6}$$

where

$$[\boldsymbol{Y}] = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{Y}_{m(1,1)} & \boldsymbol{Y}_{m(1,2)} & \cdots & 0 \\ 0 & \cdots & \boldsymbol{Y}_{m(2,1)} & \boldsymbol{Y}_{m(2,2)} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$[\boldsymbol{V}] = [{}_n\boldsymbol{V}_1 \quad {}_i\boldsymbol{V}_1 \quad \cdots \quad {}_n\boldsymbol{V}_m \quad {}_i\boldsymbol{V}_m \quad \cdots \quad {}_n\boldsymbol{V}_M \quad {}_i\boldsymbol{V}_M]^T$$

$$[\boldsymbol{I}] = [-{}_n\boldsymbol{I}_1 \quad {}_i\boldsymbol{I}_1 \quad \cdots \quad -{}_n\boldsymbol{I}_m \quad {}_i\boldsymbol{I}_m \quad \cdots \quad -{}_n\boldsymbol{I}_M \quad {}_i\boldsymbol{I}_M]^T$$

where ${}_nI_m = [I_{1_m}\ I_{2_m} \cdots I_{E_m}]$ is the current vector at group m of the SDLC, ${}_nV_m = [V_{1_m}\ V_{2_m} \cdots V_{E_m}]$ is the node voltage vector at group m of the SDLC, ${}_i\boldsymbol{I}_m$ is the branch current vector (the currents flow into the linear network) of interfacing nodes and linear subcircuit nodes at group m, ${}_i\boldsymbol{V}_m$ is the node voltage vector of interfacing nodes and linear subcircuit nodes at group m, and

$$\boldsymbol{Y}_m = \begin{bmatrix} \boldsymbol{Y}_{m(1,1)} & \boldsymbol{Y}_{m(1,2)} \\ \boldsymbol{Y}_{m(2,1)} & \boldsymbol{Y}_{m(2,2)} \end{bmatrix}$$

is the conventional indefinite nodal admittance matrix of the linear subcircuit.

Thus, the indefinite nodal admittance matrix of all of the linear subcircuits combined is a block diagonal matrix

$$\boldsymbol{Y}_L = \operatorname{diag}(\boldsymbol{Y}_1, \cdots, \boldsymbol{Y}_m, \cdots, \boldsymbol{Y}_M). \tag{7}$$

V. Augmented Admittance Matrix

To combine the linear circuits (lumped and distributed) in an augmented admittance matrix, as shown in Fig. 2, (4) is expanded into the full set of voltages $[\boldsymbol{V}]$ yielding

$$[\boldsymbol{Y_E}][\boldsymbol{V}] = [\boldsymbol{I}] \tag{8}$$

with $[\boldsymbol{Y_E}]$ being the expanded nodal representation of the SDLC, given by

$$\begin{bmatrix} {}_n\boldsymbol{Y}_{\boldsymbol{1},\boldsymbol{1}} & 0 & \cdots & 0 & {}_n\boldsymbol{Y}_{\boldsymbol{1},\boldsymbol{m}} & 0 & \cdots & 0 & {}_n\boldsymbol{Y}_{\boldsymbol{1},\boldsymbol{M}} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ {}_n\boldsymbol{Y}_{\boldsymbol{m},\boldsymbol{1}} & 0 & \cdots & 0 & {}_n\boldsymbol{Y}_{\boldsymbol{m},\boldsymbol{m}} & 0 & \cdots & 0 & {}_n\boldsymbol{Y}_{\boldsymbol{m},\boldsymbol{M}} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ {}_n\boldsymbol{Y}_{\boldsymbol{M},\boldsymbol{1}} & 0 & \cdots & 0 & {}_n\boldsymbol{Y}_{\boldsymbol{M},\boldsymbol{m}} & 0 & \cdots & 0 & {}_n\boldsymbol{Y}_{\boldsymbol{M},\boldsymbol{M}} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}.$$

Equations (7) and (8) are added together to form the overall linear circuit

$$\boldsymbol{Y_A} = \boldsymbol{Y_E} + \boldsymbol{Y_L}. \tag{9}$$

In microwave nonlinear-circuit analysis, the network parameters of the linear circuit are reduced to just include the interfacing nodes. Standard matrix-reduction techniques can be used to obtain this reduced circuit. An interfacing node is assigned to be the local reference node at each group; hence, eliminating the corresponding rows and columns. The resulting system of equations is definite and represents the augmented linear circuit.

VI. Discussion

The scheme for the augmentation of a nodal admittance matrix by a port-based matrix with a number of local reference nodes permits field-derived models to be incorporated in a general-purpose circuit simulator based on nodal formulation. The method is immediately applicable to modified nodal admittance (MNA) analysis as the additional rows and columns of the MNA matrix are unaffected by the augmentation. This work is being used in the simulation of spatial power combiners, which are electrically large and do not have a global reference node or perfect ground plane.

References

[1] A. E. Ruehli and H. Heeb, "Circuit models for three-dimensional geometries including dielectrics" *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1507–1516, July 1992.

[2] S. M. El-Ghazaly and T. Itoh, "Electromagnetic interfacing of semiconductor devices and circuits," in *Proc. IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 1, Denver, CO, June 1997, pp. 151–154.

[3] P. Ciampolini, P. Mezzanotte, L. Roselli, and R. Sorrentino, "Accurate and efficient circuit simulation with lumped-element FDTD technique," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 2207–2215, Dec. 1996.

[4] J. Kunisch and I. Wolff, "The compression approach: A new technique for the analysis of distributed circuits containing nonlinear elements," in *IEEE MTT-S Int. Microwave Symp. Workshop WSK*, Albuquerque, NM, 1992, pp. 16–31.

[5] ——, "Steady-state analysis of nonlinear forced and autonomous microwave circuits using the compression approach," *Int. J. Microwave Millimeter-Wave Computer-Aided Eng.*, vol. 5, no. 4, pp. 241–225, 1995.

[6] V. Rizzoli, A. Lipparini, and E. Marazzi, "A general-purpose program for nonlinear microwave circuit design," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 762–769, Sept. 1983.